Knots, Tangles, and Their Applications to DNA

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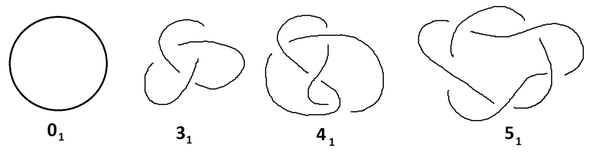
Funded by ASU Faculty Mentored Research Grant

Abstract

Mathematics is an interconnected subject with many concepts intersecting in various ways that allow for new understanding and research topics to arise. This poster gives an overview of mathematical knots, links, and tangles, and discusses the fundamental question in knot theory: how can we tell two knots apart? Some ideas used to address this question such as knot colorability and tangle embedding are outlined as well as an overview of DNA Topology and applications of knot theory to biomedical research on various diseases such as cancer and leukemia.

Knot Definitions

- A **mathematical knot** is a tangled up circle with no self-intersections.
- A **link** of n components is a collection of n knots which may or may not be interlinked.

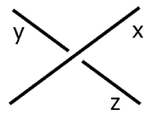


- Two knots are said to be **equivalent** if one can be transformed into the other without breaking the knot or passing it through itself.
- The fundamental question of knot theory is: **How do we tell two knots apart?**. We can use knot invariants to aid in answering this question.
- A **knot invariant** is a value or property associated to a knot that is unchanging no matter which diagram of the knot is being looked at.

Knot Colorability

Let n be a positive prime integer. Label each arc in a knot diagram with a number from the list $\{0, 1, \ldots, n-1\}$. A coloring of the diagram is said to be admissible if the following conditions hold:

1. At least two colors are used.
2. At each crossing, the colors must satisfy the condition that $2x - y - z$ is a multiple of n.



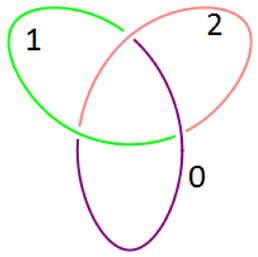
A knot or link diagram D is n-colorable, or colorable mod n if an admissible n-coloring of D exists.

Coloring Results

- A knot or link is n-colorable if it has a diagram that is colorable mod n.
- All colors present at a crossing in an admissible n-coloring of a diagram are either all the same or all different if $n > 2$.
- A knot L is not 2-colorable.

Coloring Example - The Trefoil is 3-colorable

It is easy to see that we can color each strand a different color and therefore, condition 1 is satisfied.



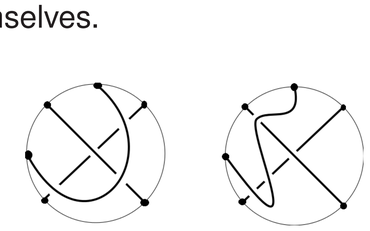
The coloring equations for the crossings are:

- $2(2) - 1 - 0 = 3 \equiv 0 \bmod 3$.
- $2(1) - 2 - 0 = 0 \equiv 0 \bmod 3$.
- $2(0) - 1 - 2 = -3 \equiv 0 \bmod 3$.

Thus, our coloring equations are all acceptable and condition 2 is satisfied. Therefore, this diagram of the trefoil is 3-colorable.

Tangle Embedding

- A $2n$-**tangle** is a collection of n arcs embedded in a solid ball so that the ends of the arcs are fixed in the boundary of the ball and the arcs do not intersect each other or themselves.

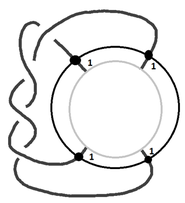


- Like knots and links, we can color tangles. Looking at this, two research questions arose.

Research Questions

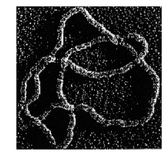
1. If T is n-colorable and T embeds in L, is L necessarily n-colorable? NO!

2. Let T be an n-colorable tangle with an admissible coloring where every boundary arc is labeled with the same color. If T embeds in a knot or link L, must L be n-colorable? YES!



DNA Topology

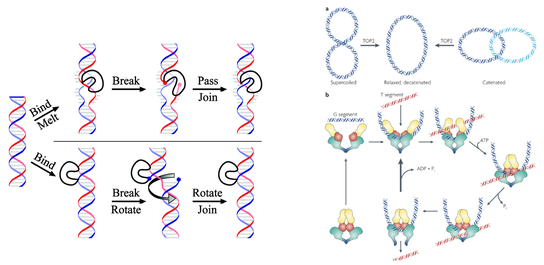
- Jerome Vinograd discovered that the double helix structure of DNA can become supercoiled and knotted. This impedes many important biological processes.



- However, this supercoiling and knotting allows scientists to experiment on cells and look at the knots that form to quantify and describe how the cell reacted.

Topoisomerases

- These are natural enzymes that can unknot and uncoil DNA in order to facilitate replication.
- Type I Topoisomerases create single-stranded breaks in the double helix then pass the opposite intact strand through the break (Type IA) or rotate the helix around the break (Type IB).
- Type II Topoisomerases generate a double-stranded break in the central axis, pass a second strand of DNA through the break, and reseal it.



Knots and Cancer

- If Type II Topoisomerase break is interrupted by a biological process, the break becomes permanent. Enough permanent breaks will kill the cell.
- Cancer cells have particularly large concentrations of Type II Topoisomerases. This allows for more breaks to occur in the cell.
- DNA-damaging agents are more effective on cells that have defective cell cycle checkpoints or defective DNA repair pathways, which cancer cells have more of than other cells.